Bar Modelling and Autism – Sufficient or Necessary in Problem solving?

Shaun Thompson

School of Education, Durham University, Durham, United Kingdom

shaun.m.thompson@durham.ac.uk

Shaun Thompson is a 4th year, part-time Ed.D. student in the School of Education, Durham University, with a research focus on the use of the bar model to support autistic pupils with mathematical problem solving.

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A significant driver for curriculum reform in England is based on performance in international comparative assessments. One consequence of this, is the rise in the use of the bar model, which is embedded within the Singapore mathematics curriculum, in mathematical problem solving. Coupled with this, is the rise in the number of pupils with autism in mainstream primary schools. This paper attempts to explore the usefulness of the bar model as a tool to support autistic pupils with mathematical problem solving. Qualitative comparative analysis is utilised in order to provide an analysis of conditions, under which the bar model may be sufficient, or necessary, to support such pupils within this domain. Findings from the study hope to support educational practitioners to maximise the teaching and learning opportunities for autistic pupils within mathematics.

Keywords: autism; bar modelling; problem solving; qualitative comparative analysis

Introduction

Significant influence on the school curriculum in the U.K. is driven by the results of international comparative assessments of academic performance such as the Programme for International Student Assessment (PISA) (DfE, 2016). Influences from those countries who demonstrate higher levels of performance than the U.K., particularly in mathematics, of which Singapore is one, frequently impact upon the classroom practice and curriculum development of schools in England. Consequently, the emphasis on mathematical reasoning and problem solving in the current National Curriculum guidance (DfE, 2013) has given rise to an increased number of schools adopting the bar model as an approach to support mathematical understanding and problem solving. Given this widespread adoption of such an approach, coupled with the fact that most autistic individuals with average or above IQ are educated in mainstream settings alongside their non-autistic peers (Bae, Chiang, & Hickson, 2015), this study seeks to address some of the gaps in current research and understanding, whilst analysing the current trends in classroom practice, in an attempt to bridge the gap between research and practice for autistic pupils. The study aims to answer the following research questions:

- Under what conditions does the bar model support mathematical problem solving, of two-step, real-life, word problems, for autistic pupils?
- Is the bar model approach sufficient to support mathematical problem solving for autistic pupils?
- Does the bar model approach form a necessary factor within a combination of other conditions to support mathematical problem solving for autistic pupils?

Mathematical problem solving and autism

Autism as a spectrum disorder

Autism spectrum disorder (ASD) acknowledges a vast heterogeneity of individuals, ranging from those with significant cognitive impairment to those with heightened cognitive abilities, compared to their neurotypical peers (Asperger's Syndrome or high functioning autism (HFA)), as well as an often uneven profile of abilities across different domains (Aagten-Murphy et al., 2013; Agrawal, 2013; Chiang & Lin, 2007; Whitby & Mancil, 2009).

Whilst acknowledging the heterogeneity of autism, various key theories have been proposed in an attempt to explain and understand the social and non-social difficulties faced by many individuals within this population. Three key theories underpinning cognition and autism are: theory of mind deficit (ToM); theory of executive dysfunction (EF); and weak central coherence theory (WCC), all of which may provide some explanation to a number of the commonly presented factors associated with autism.

An alternative to the cognitive theories discussed above, is proposed by Siegel (2009), who suggests the use of cluster deficits as a model for explaining the difficulties faced by some autistic pupils. The four clusters she considers are verbal communication, non-verbal communication, social cluster deficits and play/exploration. The use of such clusters may provide more useful to educational practitioners, as they propose a simpler mechanism by which to identify specific difficulties faced by autistic pupils.

Complexities of mathematical problem solving

The skills required to solve mathematical word problems are varied and potentially causally complex in nature. A number of processes and frameworks are required within this cognitively complex activity, including skills in linguistic interpretation, representation and computation (Bae, 2013). Through drawing on research literature, predominantly focusing on the development of mathematical problem-solving skills for autistic pupils (Aagten-Murphy et al., 2015; Bae et al., 2015; Keen, Webster, & Ridley, 2015; Wei, Christiano, Yu, Wagner, & Spiker, 2015), a complex causal diagram is presented in figure 1, below.

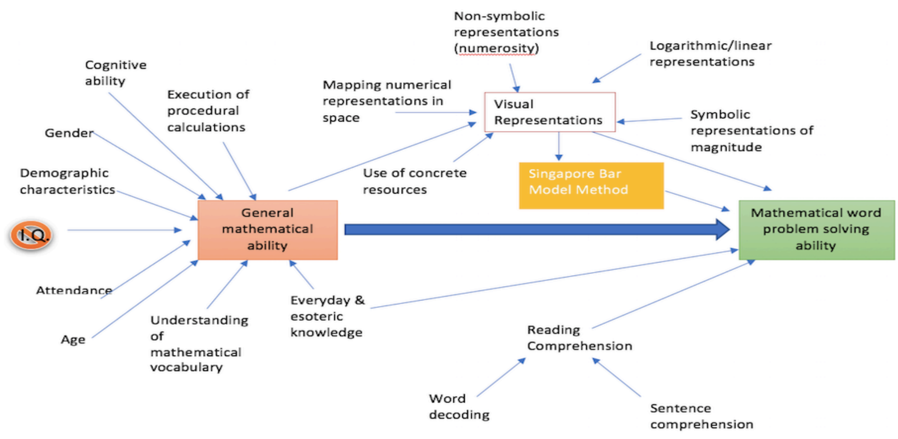

Figure 1: The complexities of mathematical problem solving

Mathematical cognition and autism

According to the research, between 6% and 22% of autistic children and adolescents are reported to struggle with number and calculation, to an extent where their maths difficulties are incommensurate with their intellectual functioning (Aagten-Murphy et al. 2015) and when it comes to mathematical problem solving, there is the requirement for the integration of several cognitive processes. However, with appropriate instruction, students with autism may have the potential to perform as well academically as their neurotypical peers.

Whilst there are widely held preconceived ideas associating exceptional mathematical abilities to the autistic population – 'savants' – frequently because of media portrayal of such individuals, for example, in the film 'Rainman', recognition of the heterogeneity of autism must be maintained (Aagten-Murphy et al., 2015). However, on the contrary, this study concluded that, on average, autistic children were 'significantly worse' than their non-autistic peers, when it came to overall mathematical achievement (p.10). Mathematical problem solving is an area where autistic individuals particularly appear to achieve disproportionately to their peers (Keen et al., 2015; Troyb et al., 2014).

Could the bar model be the solution?

The bar model approach, or a 'heuristic involving diagram or model drawing' as a tool for solving both arithmetic and algebraic word problems, is based on the theoretical framework of the processing model for solving arithmetic word problems (Kintsch & Greeno, 1985) and was officially introduced into the maths curriculum by the Singapore Ministry of Education in 1983 (Ciobanu, 2015; Ng & Lee, 2009). The aim of the bar model is to provide a consistent representational basis for the creation of a diagram that emphasises the relationships within the word problem, in order to denote a true understanding of these relationships (Maglicco & Prescott, 2016).

Application of the bar model relies on three phases: understanding the problem; the structural phase; and the procedural-symbolic phase. When considering the difficulties faced by autistic individual within mathematical problem solving, we can begin to see how these phases may support the problem-solving process for these students (figure 2), in relation to the cognitive and social theories discussed.

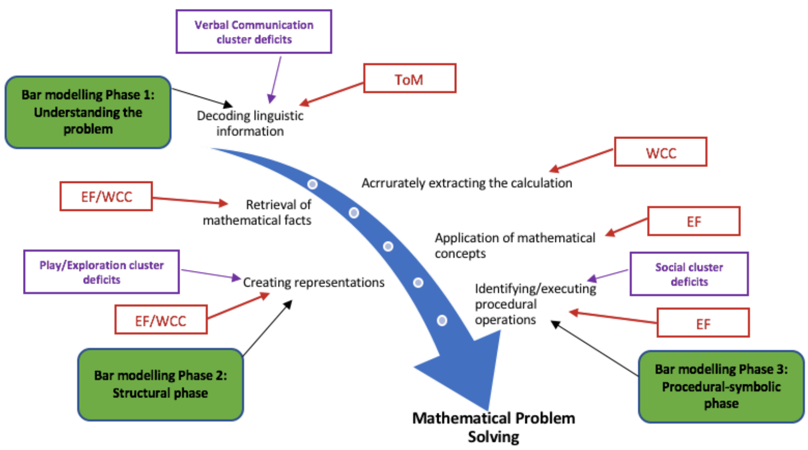

Figure 2: How deficits associated with autism may interact with the problem solving process and how the bar model may support these (adapted from Morin, Watson, Hester, & Raver, 2017).

Figure 2 begins to consider the interaction of the specific influences from cognitive and social theories of autism with the application of the bar model within mathematical problem solving. Consequently, in terms of mathematical learning and teaching, the potential for the application of the bar model approach can be tailored by practitioners to support the specific difficulties faced by individual pupils.

Theoretical basis of the bar model

Building on from the framework adopted by Kintsch and Greeno (1985), Mahoney (2012) proposed a theoretical framework which is operationalised through the bar model approach as illustrated in figure x and is based upon Mayer's two-phase model of problem solving. Central to this lies two theories – schema theory and problem solving theory (Mayer, 1989).

Schema theory, which is drawn from cognitive psychology, proposes that 'interconnected pathways within the brain are used to process and categorise new information' based on existing schemas (Maglicco & Prescott, 2016, p. 16). This theory aligns very closely with the first step in Polya's problem solving steps (Polya, 1945). It is suggested that the use of visual or schematic representations, which are central within the bar model approach, can assist with the development of new schemas and comprehension of word problems (Kintsch & Greeno, 1985; Maglicco & Prescott, 2016).

Problem solving theory is based wholly upon Polya's (1945) four-stage process to solving word problems.

Through combining these two theories, Mahoney (2012) developed a two-stage model of problem solving – problem representation stage and problem solution stage. Within this model, it is proposed that students will use their existing schemas used to solve previous word problems, in order to solve a novel problem. However, for those individuals who lack such schema, the likelihood of correctly representing the novel problem accurately, is reduced.

Within the problem representation stage, the text from the word problem is converted to an internal representation, through drawing on students' existing schemas, before being translated into an external representation by drawing on reading comprehension skills and schematic knowledge.

Students then act upon this representation during the problem-solving stage through application of appropriate algorithms, interpreted within the context of the problem. This algorithm requires the student to correctly choose the operation, calculation strategy and computation skills required to solve the problem and correctly represent the solution within the appropriate context of the problem (Maglicco & Prescott, 2016).

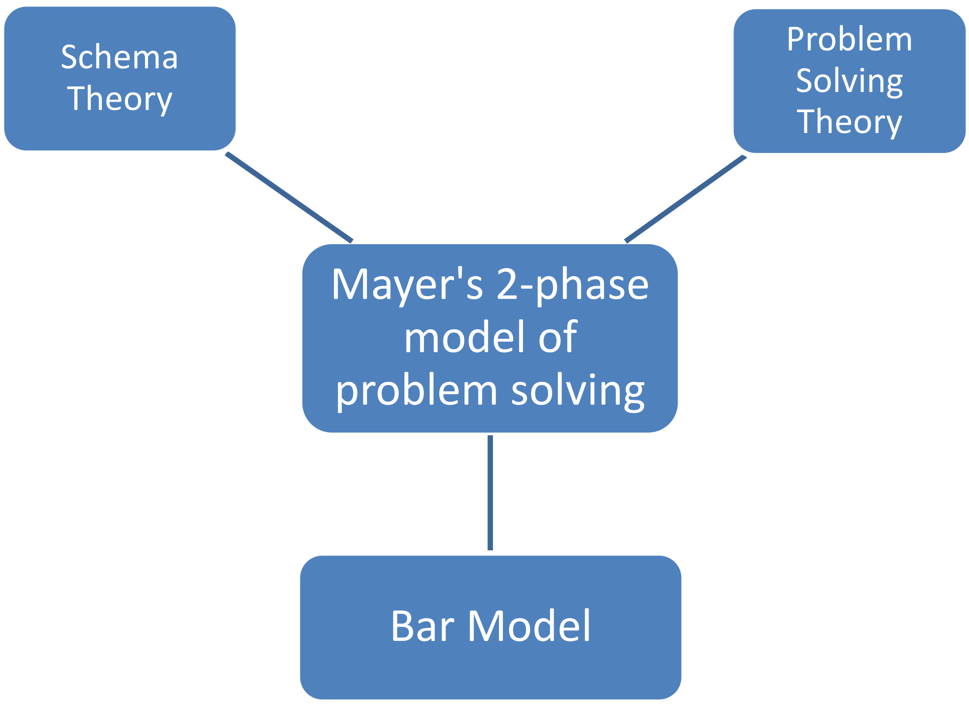

Figure 3: Mahoney's theoretical framework underpinning the bar model approach, based on Mayer's 2-phase model of problem solving (Maglicco & Prescott, 2016; Mahoney, 2012; Mayer, 1989)

According to Morin et al. (2017), this approach combines both schematic-based instruction (SBI) and cognitive strategy instruction (CSI). SBI is based on schema theory, where there is a need for students to conceptualise the underlying problem schema, and CSI involves building awareness of task demand and direct instruction of problem solving strategies, which it is suggested may address any underlying cognitive and metacognitive deficits (Morin et al., 2017).

SBI connects the two stages of Mayer's (1989) problem solving process (Maglicco & Prescott, 2016), and is based upon supporting pupils to draw upon their existing schemas in order to categorise unfamiliar word problems. As this pedagogical approach relies upon drawing on existing schemas in order to create a schematic diagram, which emphasises the underlying structure of the word problem, the consistency and fundamental simplicity of the bar model foundational structures may be key to its success. Studies have shown the success of the model approach as a tool for supporting individuals with learning difficulties (Maglicco & Prescott, 2016) and may be a direct consequence of the reduced demands of cognition and working memory required due to the consistent bar representation.

Qualitative comparative analysis (QCA) as a measure of sufficiency and necessity

Through the use of QCA, within-group differences in the autistic population can be explored, in order to ascertain potentially sufficient and necessary conditions required for problem solving within this group, an area that little research focusing on academic achievement has yet to consider (Wei, Christiano, Yu, Wagner, & Spiker, 2015). Whilst research into the problem-solving abilities of autistic pupils has been the subject of previous research, little is still known about the 'factors affecting the solution path' and more importantly, research 'within the context of any specific models or theoretical frameworks' still remains an area for exploration within this population (Bae, 2013, p. 7). In support of Bae (2013), and to strengthen the rationale behind the QCA approach to this study, Wei et al (2015) go on to state that 'factors contributing to achievement levels in autistic pupils is not well understood' and thus 'further investigation into these factors is needed' (p.201) in order to explore the 'specific kinds and combinations of interventions' required, to develop the 'applied skills and academic achievement of this population' (p.209).

Through the use of QCA, the findings from this study aim to identify the key conditions necessary for the bar model to provide a successful tool for supporting mathematical problem solving for autistic pupils, along with the conditions, under which the bar model may be a sufficient approach for mathematical problem solving.

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